Math 2A Personal Development Exercise

To be honest, it's difficult to name just one thing I never learned properly from a previous class--the years between 10 and 18 mostly saw me being terribly lost in math class. But if I had to list just ten:

a) **Trigonometric identities.** They come up at every level of math, and in different ways. There's no real general format for figuring out when they apply and how, it's just a matter of being so familiar with them you see the possible substitutions immediately, so it requires a lot of practice.

b) **Domains and ranges of various functions**, including how they change with transformations on the function or conjugation of various functions. It's not especially difficult, but useful to have a sense of upon considering any given function.

c) **Determinants.** I didn't take any classes before Math 2A at De Anza, so I can't be sure about this, but at least for me, determinants were hardly important before linear algebra and differential equations, so definitely brush up on those. Again, a very simple concept, just a matter of being comfortable enough that you don't doubt things like sign changes.

d) **Synthetic Division.** Often people get used to only dealing with linear and quadratic equations, and disregard this useful method for factoring polynomials of a higher degree. There's also a few tips to expedite the process that I definitely forgot along the way (determining the sign of the roots, using the results of incorrect guesses to eliminate other possible guesses).

e) **Properties of log/ln/exponents.** You'd think it's super easy, but people mess that stuff up more often than you'd expect. The last thing you want is to get major points off or get stuck in the middle of a problem because you couldn't remember your log rules, of all things.

f) **Partial Fractions** and **Trigonometric Substitution**. I'm combining these two because neither were covered in the calculus courses I took. The latter wasn't extremely important in Math 2A, but it's a useful technique in general, as it gives you the ability to differentiate a great many more integrals, including many of the "special" derivatives mentioned below, in the case that you blank on them. The former comes up very often in Math 2A, and you would do well to revise it before taking the class.

g) **"Special" derivatives**. You literally never know when they'll turn up, and you'll be dumbfounded and unable to continue the problem unless you remember how to take that derivative and/or integral. A lot of these can be found using trig substitution, but it is much faster if you remember a few key derivatives, and time can be everything on a test.

h) **Logarithmic Differentiation**. This relates back to the log rules from before. Know when to apply it, and how to use the properties of log and implicit differentiation correctly to evaluate the differential.

i) **Series** and Sequences. They come up more often than you'd think and are always terribly annoying to have to review. Don't do what I did. Learn it right the first time.

More than anything, the the skill that you will need the most often, on nearly every single problem, is organization. The problems only get longer and longer. Don't let yourself get confused by your own work. Don't fall prey to the all-too-common algebraic errors. Learn how to lay out your work to prevent mistakes, make checking work easy, and make it easier for your instructor to grade, giving you the maximum amount of points.

<u>Personal Development Exercise</u>

From algebra, geometry, precalculus and trigonometry:

1. Trig identities

2. Partial-fraction decomposition

3. Polish up basic algebraic skills in general

4. Factoring 3rd degree polynomials

5. Solving system of linear equations

From calculus series:

1. Derivative/antiderivative of trig functions

2. Not required for the course, but Maclaurin series

3. Integration by parts

4. Setting up parametric equations

5. Product rule

Personal Development Exercise

Tips for future math 2A students:

(a) Algebra/Pre-Cal

1) Master trigonometry identities

2) Know how to find roots of the polynomial of orders 2 and 3. Also you must know how to use imaginary numbers in case of no real roots.

3) Practice your algebra skills. Make sure you are proficient with it and make less careless error. It will affect greatly when solving math 2A's problems.

4) Know how to factor polynomials and finding the zeros.

5) Know how to use polynomial long division in case of integrating a rational function with polynomials in which the numerator has a higher order than the denominator.

(b) Calculus

1) Must be proficient with chain rule for differentiation. It will be used a lot for proving theorems and solving problems.

2) Remember the conditions for using partial fraction decomposition. It can only be used for polynomials.

3) Know your integration by parts method.

4) Must know how to manipulate power series such as the rules for adding power series together.

5) Remember your squeeze theorem. This will come out every time in calculus and math 2A class.

Dear future Math 2A students,

I'm writing to let you know about my experiences with differential equations. It should be really hard if you don't have enough preparation, but it is still not easy even with a strong foundation of calculus. Be prepared!

First of all, these are five things that I learned from the class:

- finding the zeros of polynomial by synthetic division
- calculating determinant of a matrix
- common mistakes in algebra calculation
- homogeneous properties
- substitute variables into the functions correctly

And these are the 5-not-so-obvious skills that you need to practice to do better in class:

- Integral calculation techniques (especially infinite integral)
- Calculating anti-derivatives
- Chain rule
- calculating derivatives using appropriate methods
- calculating limit using L'hospital rule

Good Luck!

Skills from Algebra, Pre-calculus, or Trigonometry:

1) Synthetic division

2) Geometric sequence

3) Complex roots ($(-1)^{(1/2)} = i$)

4) Logarithm property

5) Knowing Π (pie) : chapter 6 ➔ for simplifying the sequences sometimes we use Pie notation instead of sigma.

6) Knowing the basic trigonometry identity will help to do questions. Ex:

($\cos^2 x = (1/2)(\cos 2x + 1)$)

Skills from calculus classes:

1) Integral of secx to the power of 3.

2) Integration by part (table method)

3) Knowing the chain rule. (in section 4.7 we use chain rule, also 4.10 we use chain rule as well)

4) Partial fraction decomposition ➔ we use it a lot throughout the course

5) Hyperbolic functions (we never use these functions during the exams, but for some questions we had a chance to using them.)

6) Knowing integral and derivative of tanx , cotx , secx , $\cos^2 x$ ➔ will help a lot.

De Anza College

Math 2A

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Personal Development Exercise

Throughout the quarter, I have realized there are some skills that I think are important to succeed in this class. The first thing is you need to be really good at algebra. Because the test time is very limited, you need to solve algebra steps in the problems quickly. Secondly, you have to master in exponential/multiplication. Although it seems to be easy; however, I usually do bad on the tests because of exponential multiplication mistakes. Thirdly, you need to write equations of simple graphs. Moreover, you need to know to trigonometry identities (especially identity of cos/sin/tan/sec). Finally, you need to know how to do long division (or synthetic division) and finding roots of polynomial functions.

In calculus, you should understand the concept of continuity and differentiability. Secondly, you need to master at trigonometry differentiation/integration. I usually make mistakes on differentiating and integrating sin/cos. Thirdly, you should remember if you change the exponential of a term in a serie, how it will affect other terms and the bound of the serie. Furthermore, the class will require a lot of partial fraction integration. Finally, you need to know and master at doing chain rules in differentiation/integration. Many times I make mistakes because I forget to do the chain rules.

These concepts should be properly learned in order to succeed
- Synthetic division
- Hyperbolic trigonometric Identities
- Properties of Logarithms
- Properties of Exponents
- Partial Fraction Decompositions

These concepts should be mastered in order to attain success
- Definition of the derivative (Don't rely on just the chain rule)
- Fundamental theorems of calculus
- Substitution method for integration
- Converting from the cartesian plane to polar coordinates
- The derivatives of trigonometric functions

1. factoring polynomials is a skill I wish I had before entering this class.
2. Doing simple arithmetic without making mistakes is also a handy skill
3. I didn't know that my partial fraction decompositions were so bad
4. not knowing my simple trig angles damaged my grade
5. $e^{x+y} = e^x * e^y$

1. knowing how to do trig substitutions for integrations
2. Learn the table method For integrating by parts. It's a very useful skill. I use it all the time
3. in 2B vector projection and orthogonality came up, remember that from 1C
4. function composition, chain rule keeps coming in 1D
5. Chain rule is something I thought I would never use

5 skills from algebra, geometry, precalculus and/or trigonometry that you (or someone you know in the class) forgot or never learned properly from a previous class that caused you to do worse in Math 2A than you could have if you had mastery of them.

1. Synthetic division. This is needed for finding rational roots of a 3rd or higher degree polynomial quickly.
2. Finding complex roots. This is needed when solving homogeneous equations.
3. Recurrence relation. This is needed when finding series solutions of an ODE.
4. Organization. It is common to expect extremely long problems when solve DEs. Without organization, you could make an algebraic mistake easily and completely mess up the solution.
5. Series and sigma notation. They will appear in series solutions so you need to be comfortable with manipulating them. For example, how to shift indices.

5 not-so-obvious skills from calculus that you need to master and remember as you're progressing through the sequence Math 1A-1B-1C-1D-2A.

1. Find a limit by inspection without relying on L'hopital rule or squeeze theorem (takes too long). For example, exponential functions grow faster than polynomials, so $xe^{-x} \to 0 \ as \ x \to \infty$.
2. How to find the anti-derivative of xe^x using integration by parts and memorize the result if possible. I notice that this keeps showing up.
3. Application of integral in math 1B. This chapter teaches you how to properly set up an integral to evaluate work, force, or pressure. This skill is needed when learning 1C or 1D. In 1C and 1D there are many formulas (for example, integration in polar coordinate) that look complicated but you need to memorize them. However, if you know how to set up integrals, most of these complicated formulas are trivial and intuitive.
4. Identifying even or odd functions. You can save a lot of time if you know a function is odd of even using symmetry when evaluating integrals.
5. Product to sum and sum to product trig identity. They make trig integrations easier and faster.

Personal Development Exercise

From Algebra, Geometry, Precalculus, Trigonometry

1) Logarithmic Rules – i.e. $-4\ln x = \ln x^{-4}$, multiplication/division

2) Synthetic Division / Long Division (to simplify solutions)

3) Completing the Square (to simplify solutions)

4) Trigonometric Identities (learn to convert one form into another, or

   memorize)

   (sum & difference of cosine/sine, double angle or half-angle formula)

5) Determinant and Gaussian Elimination

From Calculus

1) Integration by Parts (be familiar with the shortcut method – saves a lot of

   time)

2) Partial Fraction Decomposition – used in Math 2A a lot

3) Product Rule – $(fg)' = f'g + fg'$

4) L'Hospital Rule & Squeeze Theorem for calculating limits

5) Chain rule – specifically changing from one dependent variable to another

   I.e. $dy/dx = (dy/dt)(dt/dx)$, $d/dx[dy/dt] = d/dt(dy/dt)dt/dx$

1. Partial fractions! → this will come up a lot in math2A.

2. Series! —

3. Differentiation & Integration!
   ↓   ↓
   Product Rule.   u-substitution.
   Chain Rule   By parts.

4. Trigonometry → Identifies such as $\sin^2 x + \cos^2 x = 1$
                                        $\sec^2 x = \tan^2 x + 1$

5. Limits!

6. Squeeze theorem.

Pre-calc...

7. Know how to find roots of order of polynomials.

8. Don't forget your imaginary numbers!

9. Some problems require you to use long division.

10. GREAT attention to detail!
    — Keep on practicing to train your brain ...

5 SKILLS FROM ALGEBRA, GEOMETRY, PRECALCULUS AND/OR TRIGONOMETRY THAT YOU (OR SOMEONE YOU KNOW IN THE CLASS) FORGOT OR NEVER LEARNED PROPERLY FROM PREVIOUS CLASS THAT CAUSED YOU TO DO WORSE IN MATH 2A THAN YOU COULD HAVE IF YOU HAD MASTERY OF THEM

1. Partial Fraction Decomposition
2. Long Division
3. Function Transformation
4. Trig. Identities (common ones)
5. Personally, imagination skill which connect numbers onto graphs and reverse.

5 NOT-SO-OBVIOUS SKILLS FROM CALCULUS THAT Y OU NEED TO MASTER AND REMEMBER AS YOU'RE PROGRESSING THROUGH THE SEQUENCE MATH 1A-1B-1C-1D-2A (DO NOT WRITE "LIMIT, DERIVATIVES, ANTI-DERIVATIVES" BECAUSE THOSE ARE OBVIOUS – YOU MAY WRITE SPECIFIC LIMIT, DERIVATIVE OR ANTI-DERIVATIVE TECHNIQUES THAT YOU DIDN'T ORIGINALLY REALIZE WOULD KEEP COMING UP OVER AND OVER AGAIN IN OTHER CLASSES)

1. Limit and derivative
   Recognization: value and limit of function, infinite and jump discontinuous, and continuous
2. Rectangualar, Parametric, Polar coordinates (including conversions from one to another)
3. Cycloid and Epic Cycloid
4. Integration of Secant to the fifth power
5. Green Theorem

Trigonometry is one thing that will always show up and very useful to memorize all identities and get really used to using them. Even hyperbolic functions are important to know.

If you are not taking 1A or 1B because you took AP calculus, you are missing a lot of information that you will need to know throughout the courses. Encourage you to take 1B although you have a credit for it, or at least study the chapters that 1A and 1B covers.

Don't forget how to integrate in various cases. You will have to study over again.

If you are a continuing student like me, or taking calculus even after few quarters of no math courses, you really want to be confident in solving prerequisite package before you move on to the chapters.

One skill that I didn't learn the way Professor Bert Lo does is substituting u and dv in integral. The way I learned was changing the formula into uv - interal(udv). It is much easier and simpler (in fact, only 1 step difference, but it makes a huge difference.) to follow Professor's method.

Hyperbolic is one thing that you might have thought that is not important, but it is good to know.

You must be familiar with trigs. I saw some of people having problem figuring out what sine or cosine is. For example, if cosine is a/b or b/a.

Knowing multiple integral learned in 1D helps a lot if you are taking Physics 4 series.

The most difficult thing for me was drawing a graph of function in my mind. Be sure you know how the graph would look like. This is implicated in everywhere you see a function. It will help you understand the questions better.

By now, you should realize that uncollected homework is not optional, but required. Do your homework a head of time, so that when your lecturer gives you a lecture, you understand clearly and ask what you don't understand when it's happening in the class.

Personal Development Exercise

A. 5 Skills from Algebra, Geometry, Precalculus, and/or Trigonometry
1. **Factoring Polynomials using Synthetic Division.** This is used when trying to find the homogenous equation to higher order differential equations.
2. **Log Rules.** You will use log (mainly ln) throughout Calculus.
3. **Trig Identities ($\sin(a+b)$, $\cos^2x = (1 - \cos2x)/2$).** You will need to know these like the back of your hand in Calculus.
4. **How to find the "equation" for a line.** This seems super basic, but I struggled when trying to do Laplace transforms on DE's where the graph of the forcing term was provided because I didn't know how to do this easily.
5. **Hyperbolic Functions.** I learned about these in Precalculus, and then completely forgot about them because my Calculus classes never used them. But, you'll need to know them in Differential Equations.

B. 5 Skills from Calculus
1. **Partial Fraction Decomposition.** You basically can't use Laplace transforms to find solutions to DE's if you don't know this.
2. **"Special" Methods of Solving Limits (L'Hospital, Squeeze).** A lot of the proofs in Differential Equations use these.
3. **How to integrate $e^x\cos x$ and $e^x\sin x$.** This is a pretty important integration that caused me to struggle on my studying a bit because I didn't know about it.
4. **Maclaurin Series.** Luckily, Mr. Lo only gave us DE's with polynomial coefficients when asked to find the series solution for them, but I can't guarantee that you'll be as fortunate.
5. **How to solve a determinant.** To prove that a differential equation is linearly independent, you need to solve a determinant named "the Wronskian".

[a] 5 skills from algebra, geometry, precalculus and/or trigonometry that are important for future classes (including **MATH 2A**)

     [1] **PARTIAL FRACTIONS!!** They can appear most of the time throughout the series Math 1A-1B-1C-1D-2A. Maybe not in 1A/1D, but definitely throughout the rest of the Math series. Learn to do them well, and take advantage of any shortcuts you learn to do these faster.

     [2] **trig identities** (ie. sum, double-angle, Pythagorean)

     [3] **solving quadratic equations** (ie. completing the square, using the quadratic formula, or factoring)

     [4] **simplifying (algebraic) expressions / combining like terms.**

         [a] Note: This is usually not forgotten, but a small mistake in doing either of these skills can make the original problem much more difficult. To do this well, you have to make sure to <u>organize your work</u>! Messy work can lead to you dropping a negative sign/ entire term, changing a 2 to a 3, etc.

     [5] **determinants**

[b] 5 not-so-obvious skills from calculus to master and remember

Skill	From Math	Also appears in	Notes
chain rule!	1A	1B, 1C, 1D, 2A	Know the definition and how to use it
Trig substitutions	1B	1C, 1D, 2A	Know how to do them well!
improper integrals	1B	1C, 2A	*In 1C:* You will see something similar to improper integrals (with proving convergence of series) *In 2A:* You will see this at the end of the quarter in Laplace Transforms (another differential equation solving technique)
L'Hôpital's rule	1A	1C, briefly in 2A	limits keep coming back… (limits are also in 1D, not L'Hôpital's rule though)
partial derivatives	1D	2A	Make sure to treat any variables, that you are not differentiating by, as constants. Apply the chain rule if necessary *In 2A:* You use this to help solve certain types of differential equations

Math2A: Personal Development Exercise

A. Algebra, geometry, precalculus and/or trigonometry:

- Rational root theorem/Descartes' rule of signs: These tricks are helpful for when you have a rather high degree polynomial that you want to factor since you can find candidates for roots just by the leading and last coefficient of it! Believe me, I was surprised to find out how important polynomials are to differential equations. Not to get too far ahead of myself, but one major method of solution is to assume $y = e^{(rx)}$ and plug it (and its derivates) into the differential equation which yields a polynomial in terms of 'r' you must solve.

- Long/synthetic division: Adding to above, if you find out one root of a polynomial (by inspection or what was described above) you can use long/synthetic division to find the rest of the factors. I remember how while I was able to do calculus on the diagnostic quiz fine, I literally couldn't factor a third-degree polynomial given on it. It was embarrassing, but then again if I can do calculus I really should be able to maintain rudimentary algebraic skills.

- Fundamental trigonometric identities: These will be handy to know because you can simplify a lot when you have trig functions together (e.g. if you have sinx and cosx in an expression you could get them into terms of just sine or cosine if its convenient). More so than identities, you should just be comfortable with trig to the point where it's intuitive to you, it makes for a great convenience.

- Properties of exponents: When working with differential equations, there are plenty of exponential expressions involved, so there's a lot of room for mistake if you don't remember the important rules of handling them.

- Properties of logarithms: When solving differential equations, you will be doing integrations and finding solutions that involve logs, so make sure you are comfortable with them (goes hand-in-hand with exponents),

B. Calculus:

- Trigonometric derivates/antiderivatives: Know them, you will working with them countless times.

- Integration by parts: This will be used plenty of times in finding solutions.

- Continuity, differentiability, intervals, etc: Review these fundamental concepts and know them well. It certainly isn't hard, but these fundamentals of calculus are important in matters such as determining the existence of solutions as well as testing for which method of solution works with a given problem.

- Substitutions: In differential equations, you will be using substitutions all the time. Get familiar with all the different substitutions you can use to ease the burden of a mathematical situation.

- Chain rule and the product rule: I note these two together with substitutions in mind, because to use a substitution correctly and consistently with differential equations you need to account for derivates involving it

P.S.: Don't forget constants when integrating (forgetting them could cost you an entire problem you conceptually understood, which you don't wanna screw up) and practice partial fraction decomposition as much as possible.

My advice to future Math 2A and calculus students would be to periodically schedule time (perhaps weekly) to study for the class aside from the homework. I would use this time to go over my lecture notes and revisit example problems from the lecture to make sure I fully understand every detail of every concept that was covered for that week. This would also prove to be helpful for the midterms and the final exam, as you would not feel as though you need to remember material you have forgotten.

Five important aspects of geometry, pre-calculus, and trigonometry that caused me to do worse in Math 2A due to forgetting them are trigonometric identities, logarithmic identities, the Rational Root Theorem, the Unit Circle, and partial fraction decomposition. Trigonometric and Logarithmic identities are very important when simplifying complicated expressions, which is useful in any math class. The Rational Root theorem was something I had to spend extra time re-learning since many equations we encounter have a degree higher than two. I also had to go back and memorize the unit circle and its corresponding sine, cosine, and tangent values in order to solve definite integrals. Learning how to break apart fractions into partial fractions is also extremely important not only when solving integrals but also when analyzing Laplace Transformations.

Five important aspects of calculus that are important to master throughout the Math 1A-1B-1C-1D-2A sequence are continuity, trig substitution for integrals, the "table method" of integration by parts, power series, and indeterminate forms. Continuity is extremely important for analyzing graphs and limits. The "table method" is an extremely short and easy way of solving integrals by parts that will save a lot of time. The understanding of power series and their convergence is extremely important when using them to solve differential equations. Indeterminate forms are also very important since we frequently encounter limits involving zero and/or infinity.

Advice to students of Math 2A – Differential Equations

Items from: Algebra, Math, Geometry

- Do refresh Binomial Expansions at least up to 3rd power → chap 7
- Partial fractions with those where the denominator is a complex root→ chap 7
- This is the most basic thing but it will cost you the most points. How to efficiently do factorizations → All chapters
- Graphical representation of functions and how they are shifted e.g. f(t-a) is f(t) shifted right → chap 7
- Relationship of sin and cosines e.g. cos (x) = sin (pi/2 - x) → chap 7

Items from: Calc 1A-1D

- Trigonometric Identities for Chapter 3
- Limits for Chap 1 (with domain of continuity)
- Integration by parts most applicable in Chp 4
- Chain Rule from 1-B for Chap 4
- Power series from 1-C for Chap 6